It seems problem 85 from section 5.5 is raising a conceptual question for some students:

If $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$

The solution shown in lecture was

Use $u-$substitution:

Let $u = 2x$

Change the limits of integration:

$x = 0 \quad \Rightarrow \quad u = 0$

$x = 2 \quad \Rightarrow \quad u = 4$

Rewrite the integrand and differential completely in terms of the new variable (u):

$$\frac{du}{dx} = 2 \;\Rightarrow\; dx = \frac{1}{2}du \;\Rightarrow\; f(2x)\,dx = f(2x)\cdot\frac{1}{2}du = \frac{1}{2}f(u)\,du$$

Rewrite the definite integral completely in terms of the new variable (u):

$$\int_0^2 f(2x)\,dx = \int_0^4 \frac{1}{2}f(u)\,du = \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\int_0^4 f(x)\,dx = \frac{1}{2}(10) = 5$$

It seems that the conceptual problem might be coming from the part: "$\frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\int_0^4 f(x)\,dx$"

Some students may be confused that it seems like

$u = 2x$ (from the $u-$substitution) and $u = x$ (from the above part)

at the same time.

This is not the case because the x's in "$\int_0^4 f(x)\,dx = 10$", "$\int_0^2 f(2x)\,dx$" and "$\frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\int_0^4 f(x)\,dx$"

are "not necessarily the same", but it doesn't matter anyway.

This last statement probably just adds to the confusion, so let me expand on the reasoning.

The central concept to keep in mind is that

the name of the variable used in a definite integral is irrelevant,
as long as its use is consistent.

That is, $\int_0^4 f(x)\,dx = \int_0^4 f(t)\,dt = \int_0^4 f(v)\,dv$.

[If you're not sure why, ask yourself what is the difference between $\int_0^4 x^2\,dx$ versus $\int_0^4 t^2\,dt$ versus $\int_0^4 v^2\,dv$.

The answer is that there is no difference since they all equal $\frac{1}{3}(4^3 - 0^3) = \frac{64}{3}$.]

So, the original question, "If $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$"

could have been written as "If $\int_0^4 f(t)\,dt = 10$, find $\int_0^2 f(2x)\,dx$" (the x's in the first integral were changed into t's).

And, in the solution, the part "$\frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\int_0^4 f(x)\,dx$"

could have been written as "$\frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\int_0^4 f(t)\,dt$" instead, and the rest of the solution would follow.

I presented this in class by changing the x's in "$\int_0^4 f(x)\,dx = 10$" and "$\frac{1}{2}\int_0^4 f(u)\,du \;=\; \frac{1}{2}\int_0^4 f(x)\,dx$" into red x's while leaving all the other x's black. My use of the red x was like changing those x's into t's as shown above.

So, in this way, the x's in "$\int_0^4 f(x)\,dx = 10$", "$\int_0^2 f(2x)\,dx$" and "$\frac{1}{2}\int_0^4 f(u)\,du \;=\; \frac{1}{2}\int_0^4 f(x)\,dx$"
are not necessarily the same,
since we could replace the x's in the first and last integrals with t's without making any difference.

In fact, we could have replaced the x's in each integral with any variables without making any difference, as long as we were consistent in using the same variable within each integral.

So, the following questions are all identical

"If $\int_0^4 f(t)\,dt = 10$, find $\int_0^2 f(2x)\,dx$"

"If $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2t)\,dt$"

"If $\int_0^4 f(\theta)\,d\theta = 10$, find $\int_0^2 f(2y)\,dy$"

"If $\int_0^4 f(y)\,dy = 10$, find $\int_0^2 f(2x)\,dx$"

and their solutions are identical to the one above, with all variables replaced accordingly.

However, by the same argument, they are all equivalent to the question

"If $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$"

which means they are all equivalent to the question

"If $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$"

with the original solution shown above and in lecture.

And this is how related integrals (like the pair in this problem) will be presented in your future classes, using the same variables in multiple integrals that look entirely different from one another.
(Look at problem 79 in section 5.5.)

And finally:

If you really understand the above, then you should understand that writing "$\frac{1}{2}\int_0^4 f(u)\,du \;=\; \frac{1}{2}\int_0^4 f(x)\,dx$" was in fact unnecessary, and the last line of the solution would have been correct just written as

$$\int_0^2 f(2x)\,dx \;=\; \int_0^4 \frac{1}{2} f(u)\,du \;=\; \frac{1}{2}\int_0^4 f(u)\,du \;=\; \frac{1}{2}(10) \;=\; 5$$